A pseudo-LISA convention for the Mock LISA Data Archive

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A first draft of pseudo-LISA conventions for the LISA orbits, for GW source objects, for the LISA TDI responses, and for the standard TDI combinations. Mostly pasted together from Refs. [2] and [7].

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I. INTRODUCTION

II. LISA ORBITS

We define the orbits of the pseudo-LISA spacecraft as defined in the Appendix of Ref. [2] (and as used in the *LISA Simulator*). Namely, in the a Solar-system–barycentric ecliptic coordinate system (SSB frame) where we have set the x axis toward the vernal point[1]. The reference orbit is defined by truncating the the exact Keplarian orbit at first order in the eccentricty e. The coordinates of each spacecraft are then given by the expressions

$$\begin{aligned} x &= a\cos(\alpha) + a\,e\left(\sin\alpha\cos\alpha\sin\beta - (1+\sin^2\alpha)\cos\beta\right), \\ y &= a\sin(\alpha) + a\,e\left(\sin\alpha\cos\alpha\cos\beta - (1+\cos^2\alpha)\sin\beta\right), \\ z &= -\sqrt{3}\,a\,e\cos(\alpha-\beta), \end{aligned} \tag{1}$$

where $\beta = 2(n-1)\pi/3 + \lambda$ ($n = 1,2,3$) is the relative orbital phase of each spacecraft in the constellation, a is the semi-major axis of the guiding center, and $\alpha(t) = 2\pi f_m t + \kappa$ is the orbital phase of the guiding center. At this order of approximation the spacecraft form a rigid equilateral triangle with sidelength $L = 2\sqrt{3}ae$. Setting $e = 0.00965$ and $a = 1$ AU yields the standard $L = 5\times 10^6$ km armlengths.

Notice that by keeping only linear terms in the eccentricity we are neglecting the variation in the optical path length that would be present if the full Keplarian orbits were used[2]. The reason for this truncation is twofold. First, it makes very little difference to the instrument response, and second, there are periodic and secular effects on the orbits from other solar system bodies (notably Earth and Jupiter) that are comparbale in size to the higher order Keplarian corrections. The precise form of the orbital perturbations will depend on when LISA is launched and the final orbital injection, so it is difficult to define a convention that is meaningful beyond leading order in e.

The parameters κ and λ set the initial location and orientation of the LISA constellation. They are related to the parameters $\bar{\phi}_0$ and α_0 used by Cutler [5] according to the mapping

$$\begin{aligned} \bar{\phi}_0 &= \kappa \\ \alpha_0 &= \frac{3\pi}{4} + \kappa - \lambda, \end{aligned} \tag{2}$$

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[1] We are using a somewhat unusual coordinate system. For objects far away, our ecliptic coordinates are very close to the standard geocentric ecliptic coordinates. However, in spirit our coordinate system is more closely related to heliocentric ecliptic coordinates, but in that case the x axis points in the direction of the heliocentric ecliptic zero, which is defined as the point where galactic equator intersects the ecliptic plane nearest to the galactic center. In contrast to the vernal equinox point, the heliocentric ecliptic zero does not precess with time. The heliocentric vernal equinox is defined by the line from the Sun to the Earth at the North Vernal Equinox. It is in the opposite direction to the geocentric vernal point. We may want to adopt heliocentric ecliptic coordinates as our default as there is little difference between barycentric and heliocentric, but it would mean adopting heliocentric longitude, which is less widely used than the geocentric system

[2] However: the LISA Simulator and Synthetic LISA actually use expressions accurate to order e^2 for the positions. Synthetic LISA uses approximate armlengths accurate to order e, while the LISA Simulator uses armlengths accurate to order e^2 and the effects of pointing ahead.

and to the paramaters η_0 and ξ_0 used by *Synthetic LISA* [4, 7] by the mapping

$$\begin{aligned} \eta_0 &= \kappa \\ \xi_0 &= 3\pi/2 - \kappa + \lambda, sw < 0 \,. \end{aligned} \tag{3}$$

The ξ_0 relation has the effect of exchanging spacecraft 2 and 3.

We should also give the conversion that maps to the Pre-Phase A report and other earlier works such as Peterseim, Jennrich & Danzmann, CQG 13, 279 (1996), Schilling, CQG 14, 1513,(1997), Peterseim, Jennrich & Danzmann, CQG 14, 1507, (1997), plus others from the 97 CQG proceedings

III. GRAVITATIONAL-WAVE SOURCES

We follow Ref. [2] (and the *LISA Simulator*) in describing the sky location of gravitational-wave sources by the unit vector $\hat{n}$,

$$\hat{n} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}\,, \tag{4}$$

(where θ and ϕ are the J2000 *ecliptic colatitude* and *longitude*, the latter measured from the vernal point, aligned with the $\hat{x}$ axis in our convention). The corresponding gravitational radiation is modeled as a plane wave in a transverse-traceless gauge, propagating in the $\widehat{\Omega} = -\hat{n}$ direction in the SSB frame. The surfaces of constant phase are then given by $\xi = t + \hat{n}\cdot\mathbf{x} = \text{const}$. A generic gravitational wave can be decomposed into two standard polarization states,

$$\mathbf{h}(\xi,\hat{n}) = h_+(\xi)\mathbf{e}^+(\hat{u},\hat{v}) + h_\times(\xi)\mathbf{e}^\times(\hat{u},\hat{v})\,, \tag{5}$$

where $\mathbf{e}^+$ and $\mathbf{e}^\times$ are the polarization tensors

$$\begin{aligned} \mathbf{e}^+ &= \hat{u}\otimes\hat{u} - \hat{v}\otimes\hat{v}, \\ \mathbf{e}^\times &= \hat{u}\otimes\hat{v} + \hat{v}\otimes\hat{u}, \end{aligned} \tag{6}$$

and where

$$\begin{aligned} \hat{u} &= \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}, \\ \hat{v} &= \sin\phi\,\hat{x} - \cos\phi\,\hat{y}\,. \end{aligned} \tag{7}$$

If we refer gravitational-wave emission to the *principal polarization axes* $\hat{p}$ and $\hat{q}$ of the source,

$$\mathbf{h}(\xi,\hat{n}) = h^S_+(\xi)\,\boldsymbol{\epsilon}^+(\hat{p},\hat{q}) + h^S_\times(\xi)\,\boldsymbol{\epsilon}^\times(\hat{p},\hat{q}), \tag{8}$$

with

$$\begin{aligned} \boldsymbol{\epsilon}^+ &= \hat{p}\otimes\hat{p} - \hat{q}\otimes\hat{q}, \\ \boldsymbol{\epsilon}^\times &= \hat{p}\otimes\hat{q} + \hat{q}\otimes\hat{p}\,. \end{aligned} \tag{9}$$

we can go back to the general decomposition (5) by setting

$$h_+(\xi) = \cos(2\psi)\,h^S_+(\xi) + \sin(2\psi)\,h^S_\times(\xi), \tag{10}$$

$$h_\times(\xi) = \cos(2\psi)\,h^S_\times(\xi) - \sin(2\psi)\,h^S_+(\xi), \tag{11}$$

where $\psi = -\arctan(\hat{v}\cdot\mathbf{p}/\hat{u}\cdot\mathbf{p})$ is the *source polarization angle*. For a binary system, the inclination angle ι is defined as the angle between the line of sight $\hat{n}$ and the orbital angular momentum vector of the binary $\mathbf{L}$, so that $\iota = \arccos(\hat{L}\cdot\hat{n})$.

Put in the relation between angle to the line of ascending nodes of a binary system and the polarization angle. Given in the paper by Whalquist

These variables are related to the set $(\theta_s, \phi_s, \theta_L, \phi_L)$ used by Cutler [5] by

$$\begin{aligned} \theta &= \theta_s \\ \phi &= \phi_s \\ \iota &= \arccos\left(cos\theta_L\cos\theta_s + \sin\theta_L\sin\theta_s\cos(\phi_s-\phi_L)\right) \\ \psi &= \arctan\left(\frac{\cos\theta_s\sin\theta_L\cos(\phi_s-\phi_L) - \cos\theta_L\sin\theta_s}{\sin\theta_L\sin(\phi_s-\phi_L)}\right)\,, \end{aligned} \tag{12}$$

and to the set $(beta, \lambda, \psi, \iota)$ used in *Synthetic LISA* [4, 7] by

$$\begin{aligned} \theta &= \frac{\pi}{2} - \beta \\ \phi &= \lambda \\ \iota &= \iota \\ \psi &= -\psi_{\mathrm{SL}} \end{aligned} \tag{13}$$

Here β is the J2000 *ecliptic latitude*, and ψ_{SL} is just called ψ in the *Synthetic LISA* literature.

IV. LISA RESPONSES

The basic LISA response to gravitational waves is taken to be the *phase response* Φ_{ij} used in the *LISA Simulator* and discussed in Sec. II of Ref. [2] [see especially Eqs. (4)–(13) and (22)] or equivalently the *fractional frequency response* y^{gw}_{slr} used in *Synthetic LISA* and discussed in Sec. II B of Ref. [7] (i and s identify the transmitting spacecraft, j and r the receiving spacecraft for each phase measurement, l is a redundant link index).

The phase and fractional frequency formalisms are equivalent, and related by a simple time integration. It is not clear at this time which will be the primary format for LISA data, and perhaps both should be adopted concurrently. The frequency measurements have the advantage of being directly proportional to the gravitational strain; the phase measurements have the advantage of representing more closely the actual output of the LISA phasemeters.

V. TDI OBSERVABLES

At present it appears that Time Delay Interferometry[8] will be needed to cancel laser phase noise (arm locking may soften the requirements, but is unlikely to dispense with the need for TDI).

We will adopt the modified Time Delay Interferometry variables (TDI 1.5) [9, 10], as defined below, as the standard pseudo-LISA data outputs. The modified TDI variables are a nice compromise between the unrealistically simple Michelson variables that are swamped by laser phase noise, and the complicated second generation TDI variables that are designed to cancel laser phase noise in an array that both rotates and flexes. The modified TDI variables fit nicely with the order e trunctation of the spacecraft orbits, as the TDI 1.5 scheme is able to account for rotation but not flexing[9].

We define the standard TDI observables following the *Synthetic LISA* [4, 7] naming scheme and sign conventions (see also the *Synthetic LISA* file `lisasim-tdi.cpp`). All of these can be used both as frequency and phase observables by replacing y_{slr} measurements with Φ_{ij} measurements. See the TDI Rosetta Stone [6] for translations between index notations (in particular, the primed indices of Ref. [10] correspond to positive indices in the *Synthetic LISA* usage).

- First-generation TDI (TDI 1.0): the *Sagnac* observables α, β, γ ("centered", respectively, on spacecraft 1, 2, 3, as all following sets of three), and the *symmetrized Sagnac* observable ζ, as defined in Ref. [8]. No need to define the eight-pulse observables (Michelson, etc.), which are the same as in modified TDI.

- Modified TDI (TDI 1.5): the *unequal-arm Michelson* observables X, Y, Z; the *relay* observables U, V, W; the *monitor* observables E, F, G; the *beacon* observables P, Q, R; the *Sagnac* observables α_1, α_2, α_3; and the *symmetrized Sagnac* observables ζ_1, ζ_2, ζ_3 as defined in Ref. [10].

- Second-generation TDI (TDI 2.0): the *unequal-arm Michelson* observables X_1, X_2, X_3; the *relay* observables U_1, U_2, U_3; the *monitor* observables E_1, E_2, E_3; the *beacon* observables P_1, P_2, P_3 as defined in Ref. [10].

- Optimal TDI observables: in first-generation TDI, A, E, and T as defined in terms of α, β, γ in Ref. [11]; in second-generation TDI, $\bar{A}$, $\bar{E}$, $\bar{T}$ as defined in terms of α_1, α_2, α_3 in Ref. [12].

Note also that there is a naming conflict here between the first-generation spacecraft-1-centered monitor observable and the first-generation spacecraft-2-centered optimal observable.

VI. CONCLUSIONS

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